Discrete Fourier Transform (DFT)

Inverse DFT
Transform Fourier coefficients to time domain samples

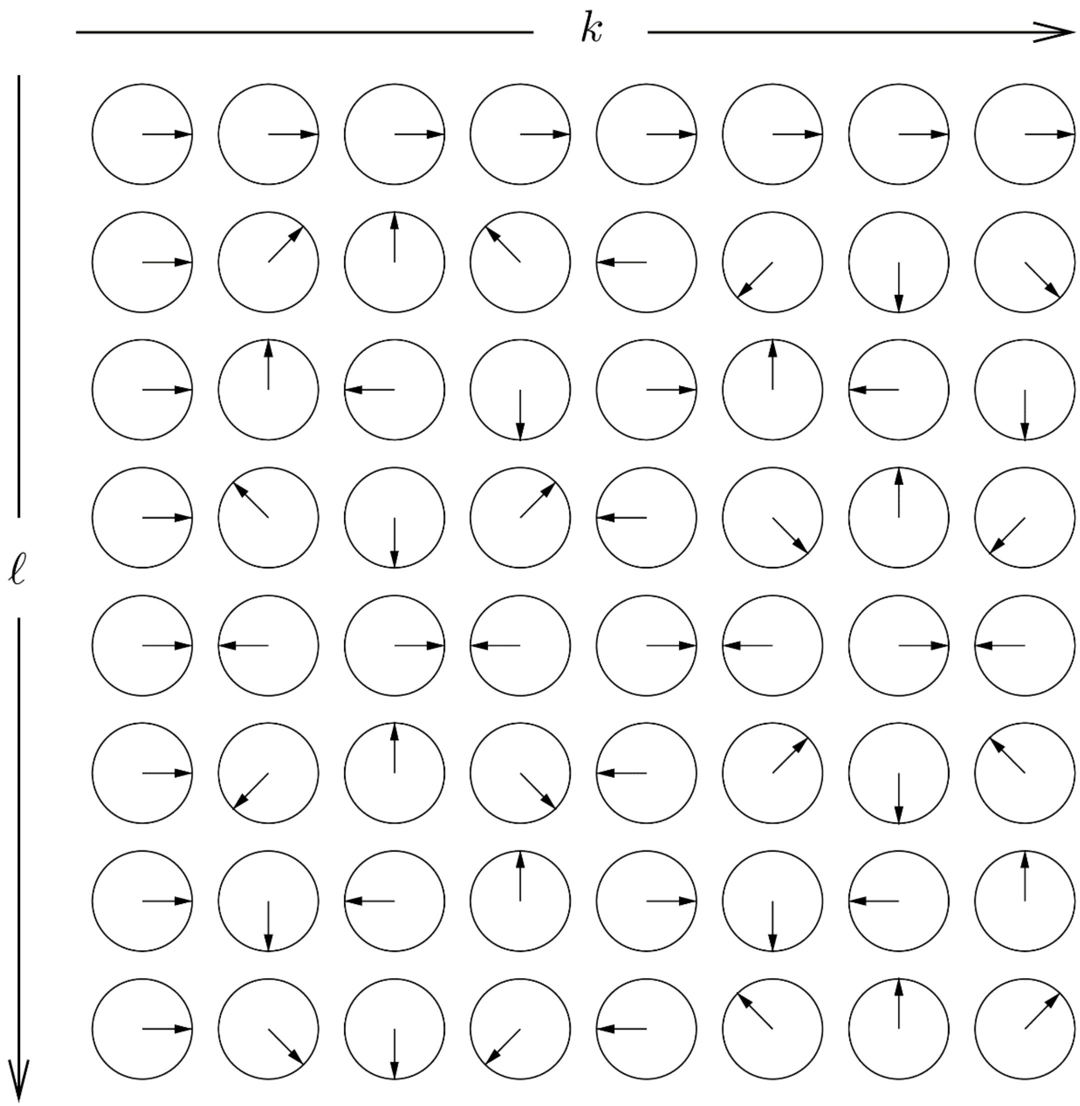
$$\begin{aligned}
\vec{f} &= B\vec{z} \\
f_\ell &= \sum_{k=0}^{n-1} B_{\ell k} z_k = \sum_{k=0}^{n-1} z_k e^{2\pi j k\ell/n}, \qquad \ell = 0, \ldots, n-1
\end{aligned}$$

DFT
Transform time domain samples to Fourier coefficients

$$\begin{aligned}
\vec{z} &= \frac{1}{n} B^* \vec{f} \\
z_k &= \frac{1}{n} \sum_{\ell=0}^{n-1} B^*_{k\ell} f_\ell = \frac{1}{n} \sum_{\ell=0}^{n-1} f_\ell e^{-2\pi j k\ell/n}, \qquad k = 0, \ldots, n-1
\end{aligned}$$

Matrix B for $n = 8$

$$b_{\ell k} = e^{2\pi j k \ell / 8}$$
$$|b_{\ell k}| = 1$$



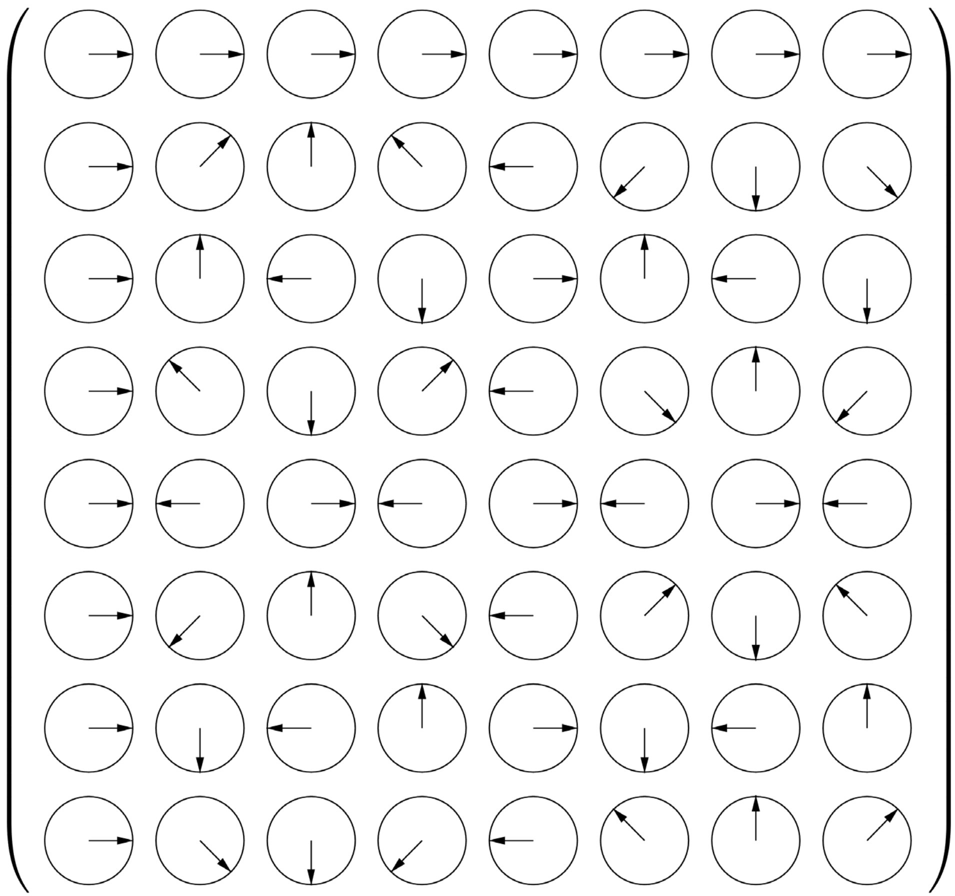
$e^{2\pi j k 0/8}$, all angles 0

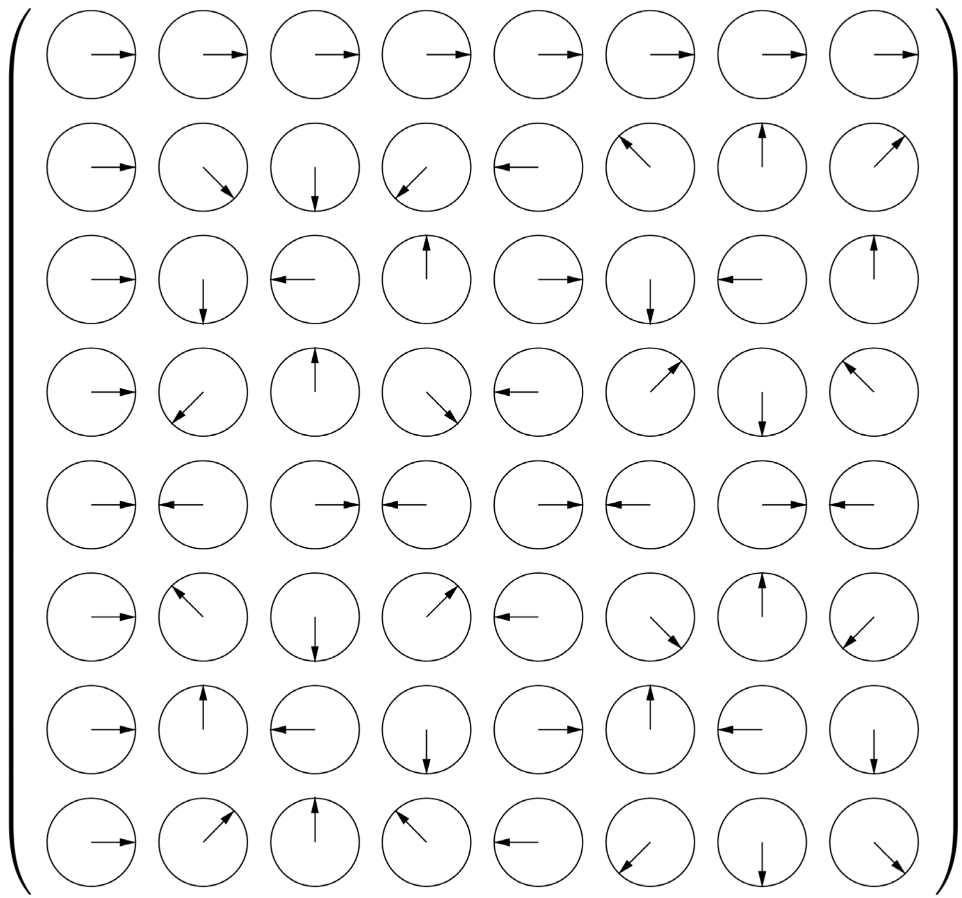
$e^{2\pi j k 1/8}$, angle steps $\pi/4$

$e^{2\pi j k 2/8}$, angle steps $\pi/2$

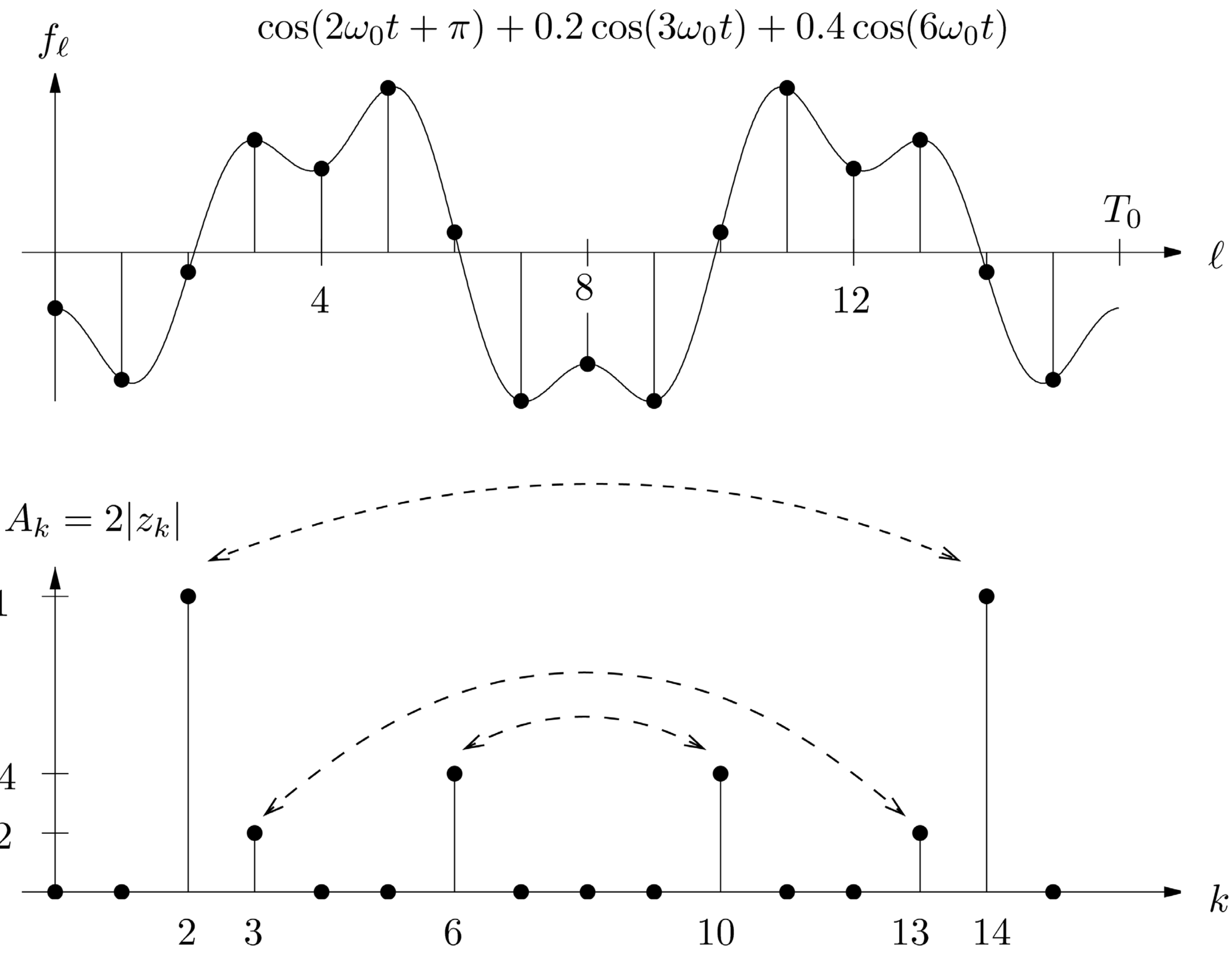
$e^{2\pi j k 4/8}$, angle steps π

$B =$



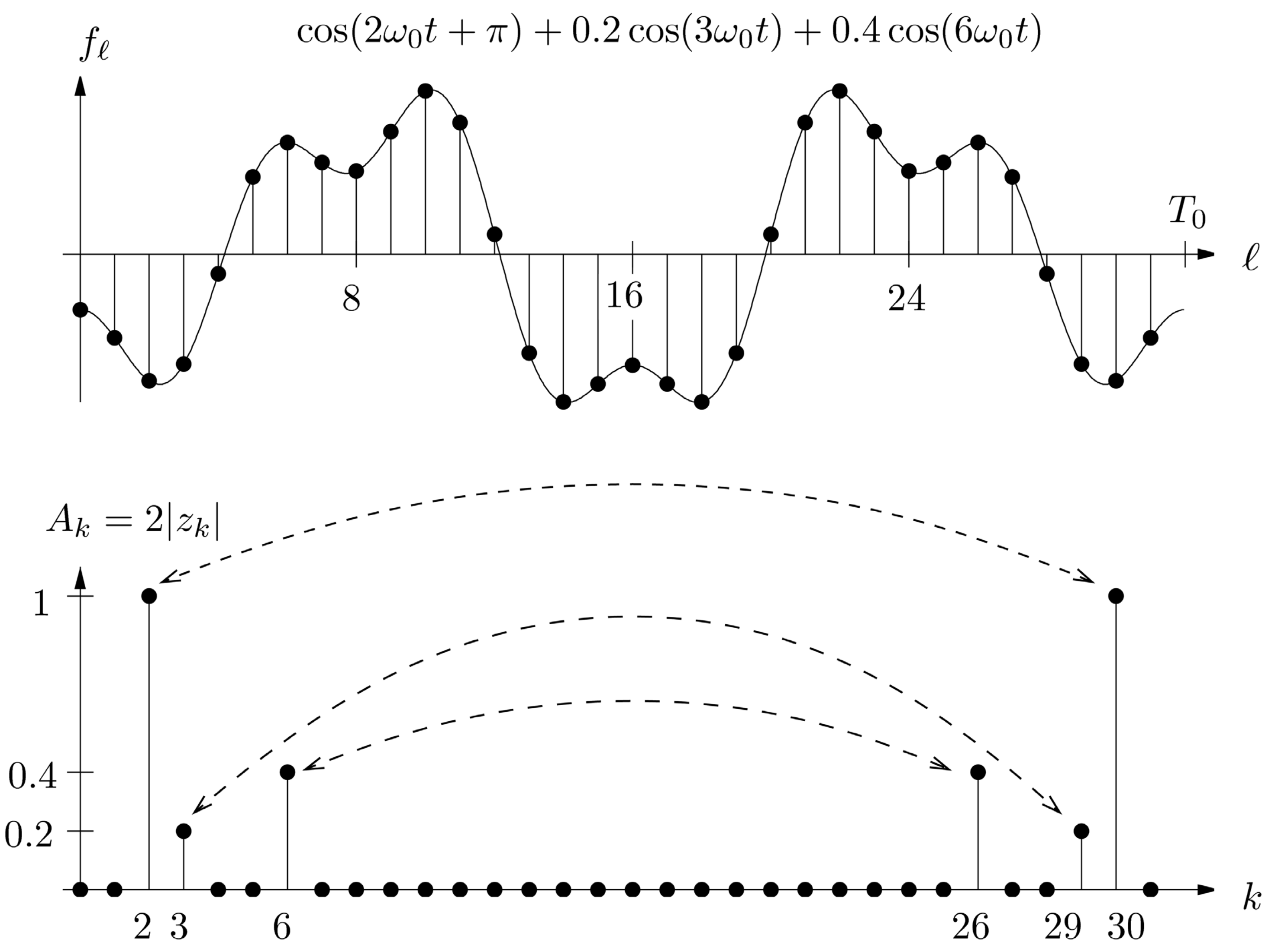
$B^* =$ 

n=16

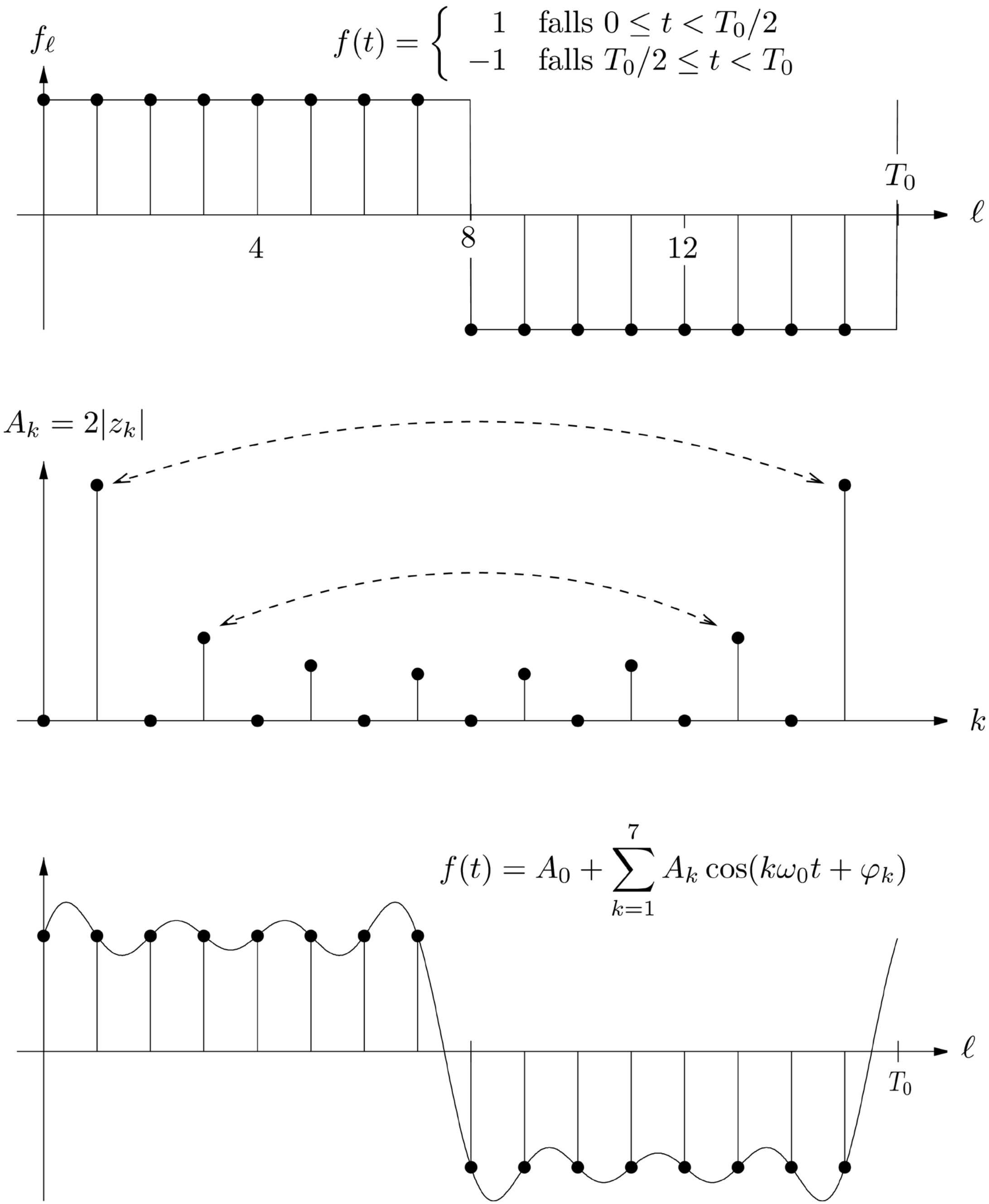


$\cos(2\omega_0 t + \pi) + 0.2\cos(3\omega_0 t) + 0.4\cos(6\omega_0 t)$
f_ℓ
T_0
ℓ
4
8
12
$A_k = 2|z_k|$
1
0.4
0.2
k
2
3
6
10
13
14

n=32

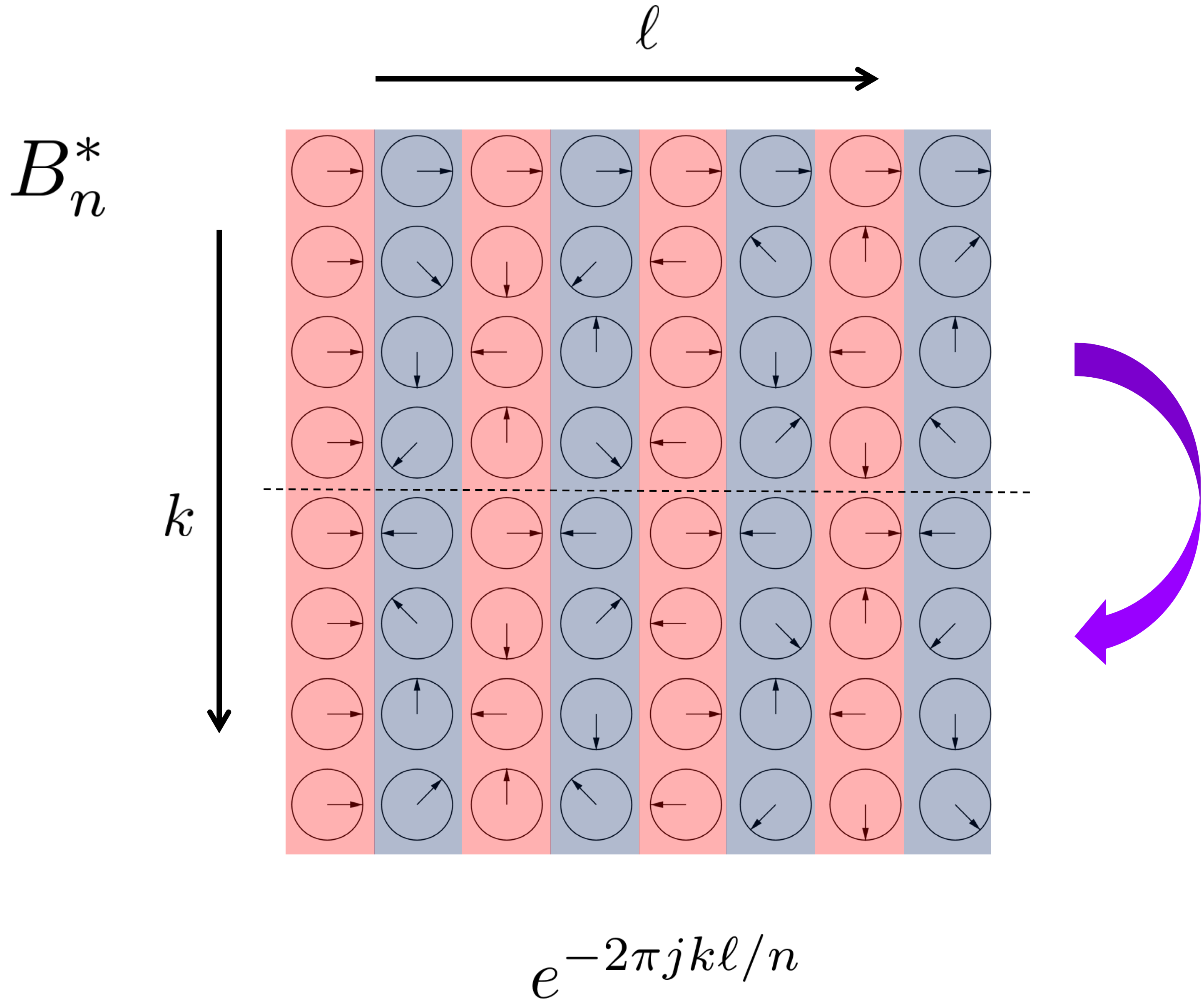

$\cos(2\omega_0 t + \pi) + 0.2\cos(3\omega_0 t) + 0.4\cos(6\omega_0 t)$
f_ℓ
T_0
ℓ
8
16
24
$A_k = 2|z_k|$
1
0.4
0.2
k
2
3
6
26
29
30

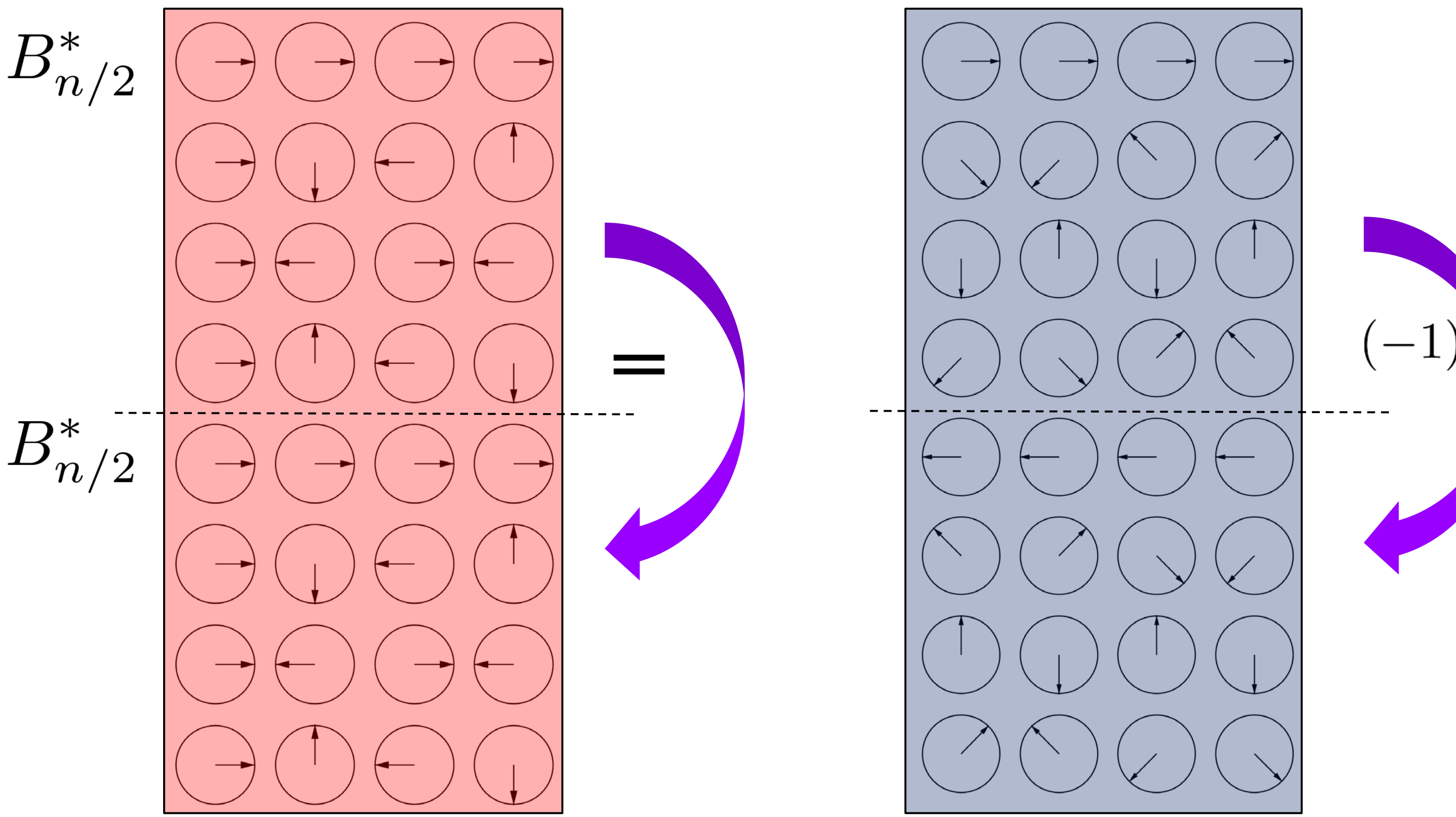
n=16



f_ℓ
$f(t) = \begin{cases} 1 & \text{falls } 0 \le t < T_0/2 \\ -1 & \text{falls } T_0/2 \le t < T_0 \end{cases}$
T_0
ℓ
4
8
12
$A_k = 2|z_k|$
k
$f(t) = A_0 + \sum_{k=1}^{7} A_k \cos(k\omega_0 t + \varphi_k)$
ℓ
T_0

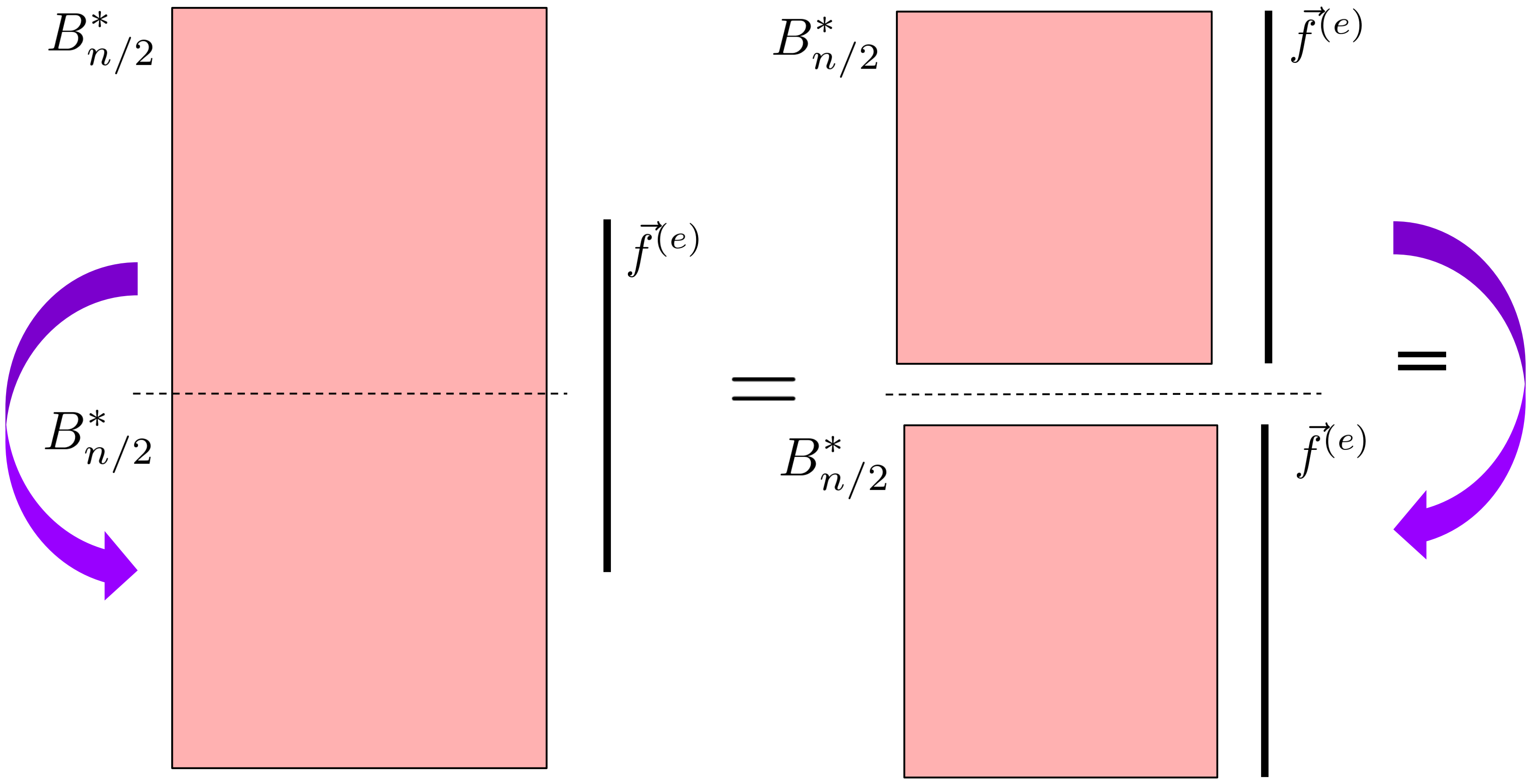
Fast Fourier Transform (FFT)


ℓ
B_n^*
k
$e^{-2\pi jk\ell/n}$

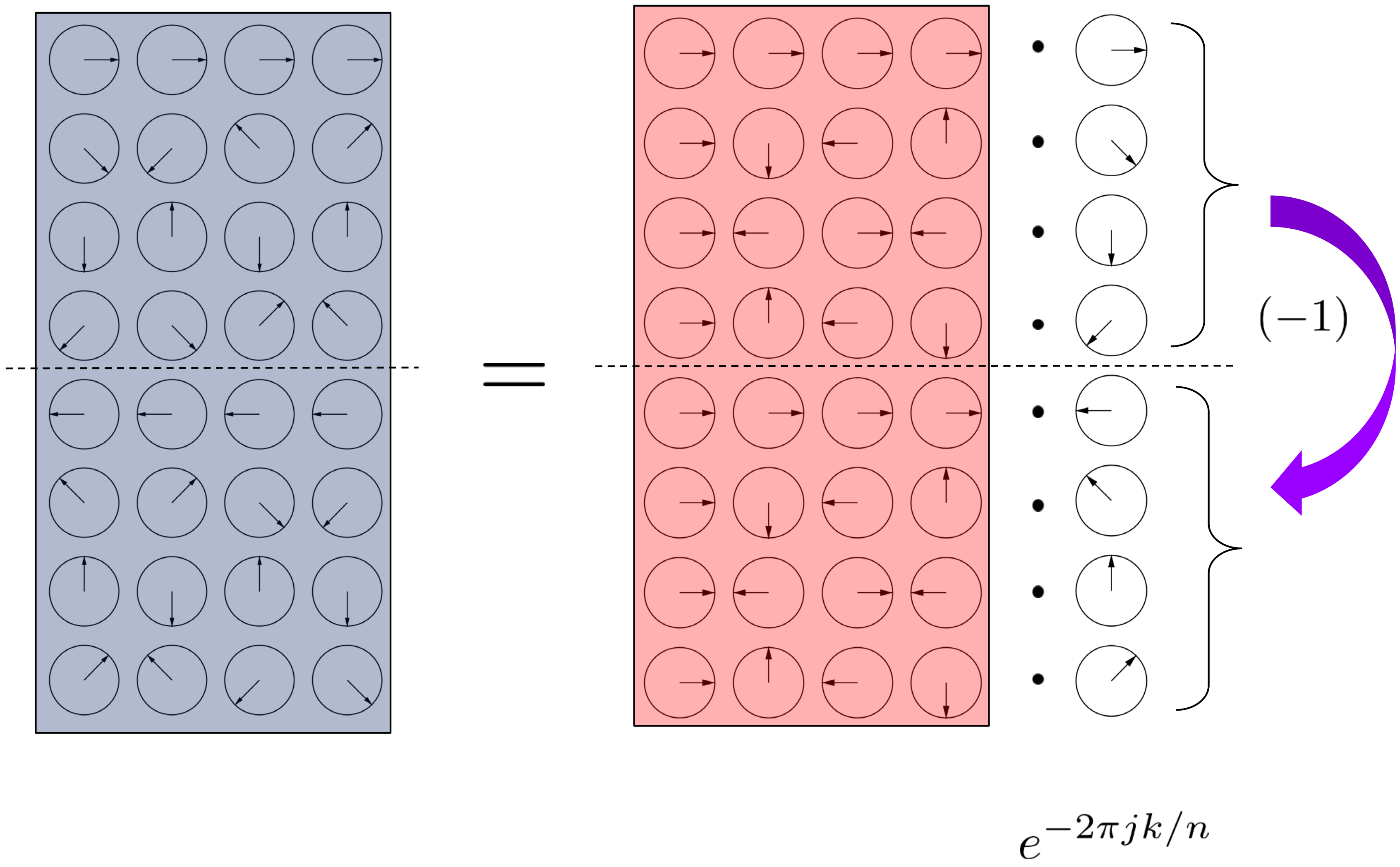


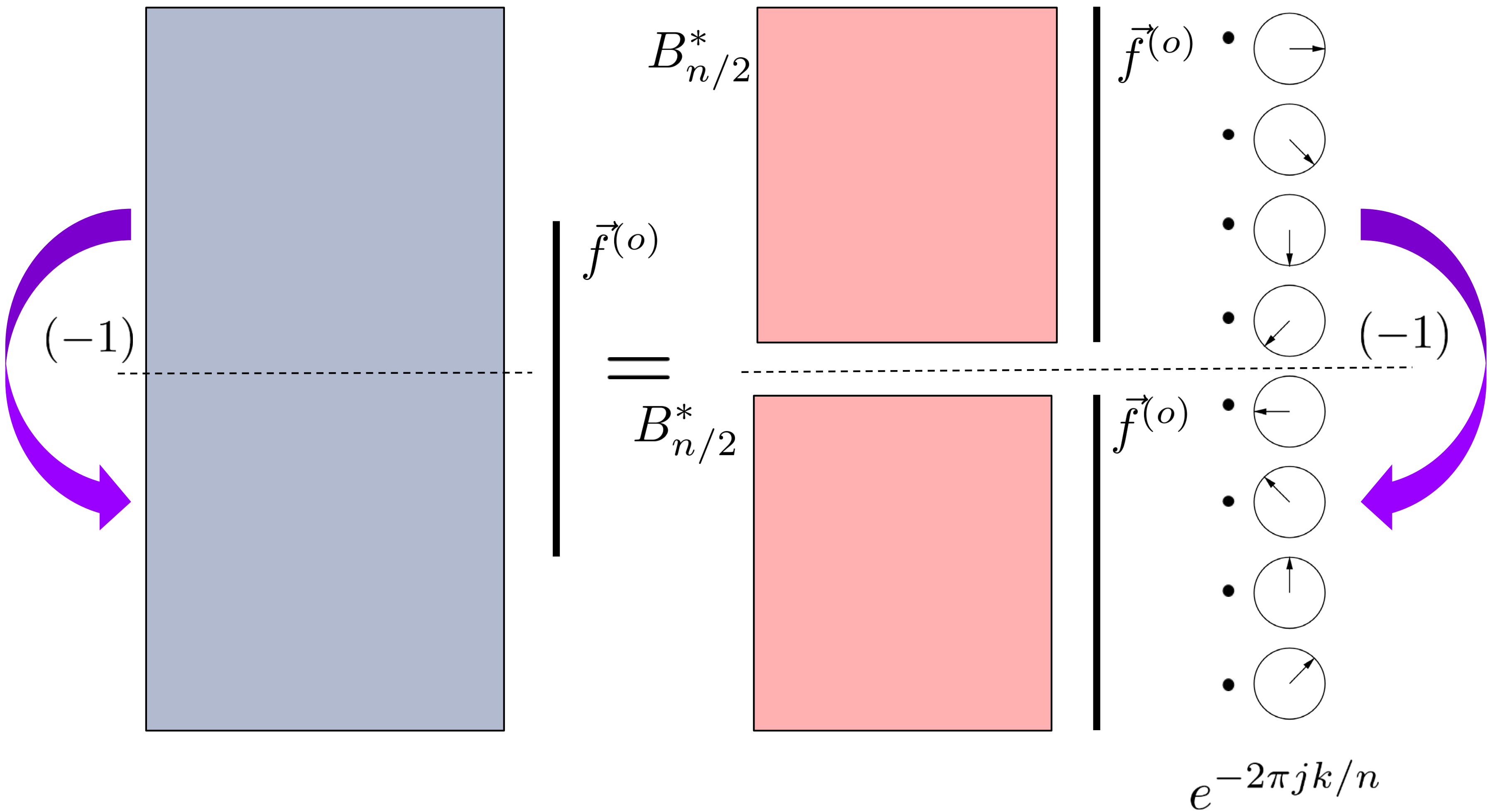
$$e^{-2\pi jk2\ell/n} = e^{-2\pi jk\ell/(n/2)}$$

$$e^{-2\pi jk(2\ell+1)/n} = e^{-2\pi jk\ell/(n/2)}e^{-2\pi jk/n}$$



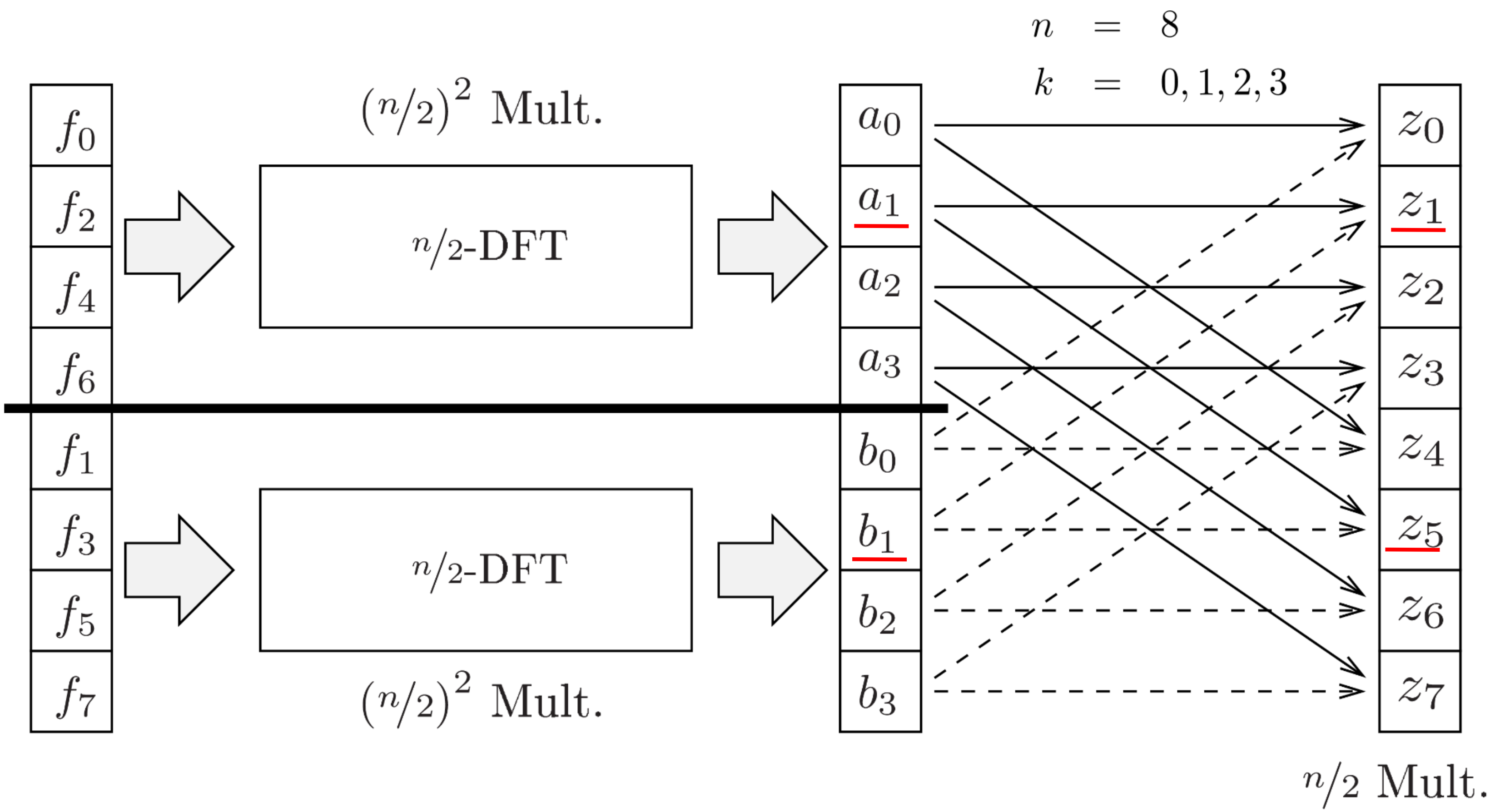
Reduction of effort by half!

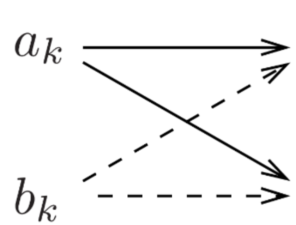

(−1)
$e^{-2\pi jk/n}$



Reduction of effort by half!

Order 8-DFT reduced to 2 order 4-DFTs

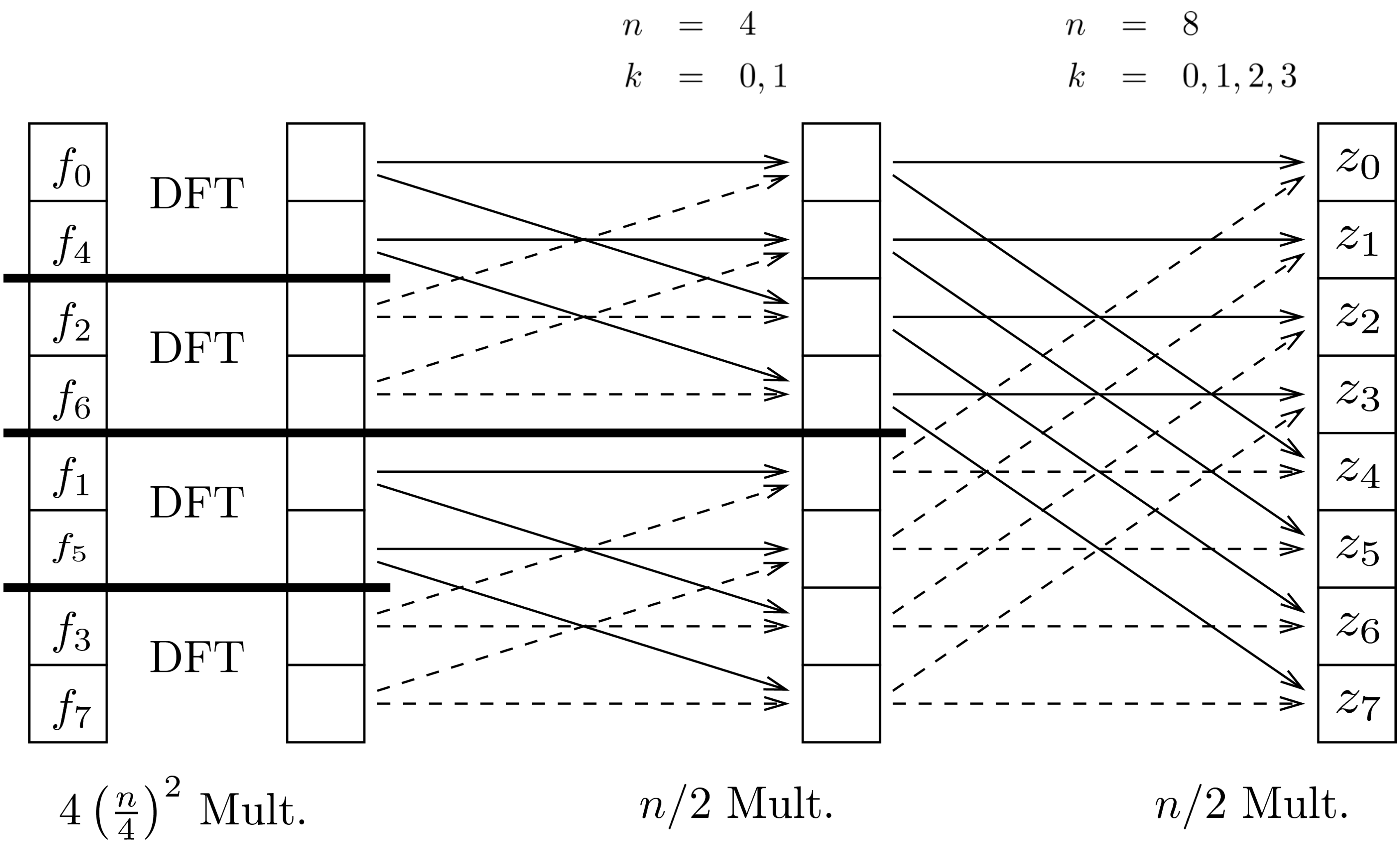




$$z_k = a_k + b_k e^{-2\pi jk/n}$$

$$z_{k+n/2} = a_k - b_k e^{-2\pi jk/n}$$

2 order 4-DFTs reduced to 4 oder 2-DFTs



4 order 2-DFTs reduced to 8 oder 1-DFTs: FFT.