Heilbronn, 7.10.2025 Prof. Dr. V. Stahl WS 25/26

Homework for Digital Signal Processing with Solutions

Sheet 2

Exercise 1. Implement the following functions (matlab is easiest):

- A function which reads a sound file (WAV format) and writes the samples (one channel in case of stereo) to a text file.
- A function which reads samples from a text file and stores the result in a sound file (WAV format, mono, 8kHz sampling rate).
- A function which plots the samples of a sound file.

If you decide to do your project in C, you can leave the sound file I/O to matlab and read/write samples from/to simple text files. It is also not difficult to implement sound file I/O directly in C if you make yourself familiar with the WAV header and ask Chat GPT for an example.

Finally write a program in the language for your project which generates samples for a sine wave with 200Hz and 8kHz sampling rate and duration one second. Store the samples in a sound file in WAV format. Play the sound file and verify that it sounds correct. Try different frequencies and see what happens if the frequency is above 4kHz.

Solution for Exercise 1. Programming exercise.

Exercise 2. Find a complex number z such that

$$3\cos(2t+1) = \mathrm{re}(ze^{j\,2t}).$$

Solution for Exercise 2.

$$\begin{aligned} 3\cos(2t+1) &= \mathrm{re}(3e^{j(2t+1)}) \\ &= \mathrm{re}(3e^{j}e^{j\,2t}) \\ &= \mathrm{re}(ze^{j\,2t}) \end{aligned}$$

for $z = 3e^{j}$.

Exercise 3. Find $a, b \in \mathbb{R}$ such that

$$3\cos(2t+1) = a\cos(2t) + b\sin(2t).$$

Solution for Exercise 3.

$$\begin{aligned} 3\cos(2t+1) &= 3\mathrm{re}(e^{j(2t+1)}) \\ &= 3\mathrm{re}(e^{j}e^{j\,2t}) \\ &= 3\mathrm{re}((\cos(1) + j\sin(1))(\cos(2t) + j\sin(2t))) \\ &= 3(\cos(1)\cos(2t) - \sin(1)\sin(2t)) \\ &= 3\cos(1)\cos(2t) + (-3\sin(1))\sin(2t) \\ &= a\cos(2t) + b\sin(2t) \end{aligned}$$

for $a = 3\cos(1)$ and $b = -3\sin(1)$.

Exercise 4. A function $f(t)$ is T-periodic if

$$f(t+T) = f(t) \text{ for all } t.$$

Show that

$$f(t) = e^{j\omega t} \text{ with } \omega = \tfrac{2\pi}{T}$$

is a T-periodic function.

Solution for Exercise 4. We have to show that

$$e^{j\omega(t+T)} = e^{j\omega t} \text{ for all } t.$$

As $\omega T = 2\pi$ and $e^{2\pi j} = 1$ it holds that

$$\begin{aligned} e^{j\omega(t+T)} &= e^{j\omega t}e^{j\omega T} \\ &= e^{j\omega t}e^{j2\pi} \\ &= e^{j\omega t}. \end{aligned}$$

Exercise 5. Show that the Fourier coefficients z_k of a T-periodic function $f(t)$ can also be computed with the following formula:

$$z_k = \int_0^1 f(Tt)e^{-2\pi jkt}dt.$$

Show that the function

$$g(t) = f(Tt)$$

which appears in this formula, is 1-periodic.

Solution for Exercise 5. The standard formula for computing of Fourier coefficients is

$$z_k = \frac{1}{T}\int_0^T f(t)e^{-jk\omega t}dt.$$

Substitution

$$\begin{aligned} u &= \frac{t}{T} \\ \frac{du}{dt} &= \frac{1}{T} \\ dt &= T du \end{aligned}$$

gives

$$\begin{aligned} z_k &= \frac{1}{T}\int_0^1 f(Tu)e^{-jk\omega Tu}T du \\ &= \int_0^1 f(Tu)e^{-2\pi jku}du. \end{aligned}$$

Substitution

$$\begin{aligned} t &= u \\ dt &= du \end{aligned}$$

gives

$$z_k = \int_0^1 f(Tt)e^{-2\pi jkt}dt.$$

The function

$$g(t) = f(Tt)$$

is 1-periodic:

$$\begin{aligned} g(t+1) &= f(T(t+1)) \\ &= f(Tt+T) \\ &= f(Tt) \\ &= g(t) \end{aligned}$$

as $f(t)$ is a T-periodic function.

Exercise 6. Compute the complex Fourier coefficients z_k of the T-periodic function f, defined as

$$f(t) = \begin{cases} 0 & \text{for } -T/2 < t < 0 \\ 1 & \text{for } 0 \le t \le T/2 \end{cases}$$

and $f(t+T) = f(t)$ for all t. Solve the integrals without a computer. Hint: You have to distinguish the cases k even and k odd.

Solution for Exercise 6. Let

$$\omega = \frac{2\pi}{T}.$$

Then

$$\begin{aligned} z_k &= \frac{1}{T}\int_0^T f(t)e^{-jk\omega t}dt \\ &= \frac{1}{T}\int_0^{T/2} e^{-jk\omega t}dt. \end{aligned}$$

For the special case $k = 0$ we obtain

$$z_0 = \frac{1}{T}\int_0^{T/2} 1dt = \frac{1}{2}.$$

For $k \neq 0$ we obtain

$$\begin{aligned} z_k &= \frac{1}{-jk\omega T}\left[e^{-jk\omega t}\right]_0^{T/2} \\ &= \frac{j}{k\omega T}\left(e^{-jk\omega T/2} - 1\right) \qquad \omega = 2\pi/T \\ &= \frac{j}{2\pi k}\left(e^{-jk\pi} - 1\right) \\ &= \frac{j}{2\pi k}(\cos(k\pi) - j\underbrace{\sin(k\pi)}_{=0} - 1) \\ &= \frac{j}{2\pi k}(\cos(k\pi) - 1). \end{aligned}$$

For further simplification we have to distinguish the cases k even and k odd:

- k even.

$$\frac{j}{2\pi k}\big(\cos(k\pi) - 1\big) = \frac{j}{2\pi k}(1-1) = 0$$

- k odd.

$$\frac{j}{2\pi k}\big(\cos(k\pi) - 1\big) = \frac{j}{2\pi k}(-1-1) = -\frac{j}{\pi k}$$

Summarizing we obtain

$$z_k = \begin{cases} 1/2 & \text{if } k = 0 \\ 0 & \text{if } k \text{ even, } k \neq 0 \\ -j/(\pi k) & \text{if } k \text{ odd} \end{cases}$$

Exercise 7. Let $f(t)$ be a T-periodic function with given Fourier coefficients z_k. The function

$$g(t) = f(t - \hat{t})$$

is $f(t)$ shifted by $\hat{t}$ to the right and therefore also T-periodic. Compute the Fourier coefficients of $g(t)$ in dependence of z_k.

Solution for Exercise 7. The Fourier coefficients of $g(t)$ are by definition

$$\frac{1}{T}\int_0^T g(t)e^{-j\omega t}dt = \frac{1}{T}\int_0^T f(t-\hat{t})e^{-j\omega t}dt.$$

Substitution

$$\begin{aligned} u &= t - \hat{t} \\ du &= dt \\ t &= u + \hat{t} \end{aligned}$$

gives

$$\frac{1}{T}\int_{-\hat{t}}^{T-\hat{t}} f(u)e^{-j\omega(u+\hat{t})}du = e^{-j\omega\hat{t}}\frac{1}{T}\int_{-\hat{t}}^{T-\hat{t}} f(u)e^{-j\omega u}du.$$

As the integrand is T-periodic and we integrate over one period, we can simplify as

$$e^{-j\omega\hat{t}}\frac{1}{T}\int_0^T f(u)e^{-j\omega u}du = e^{-j\omega\hat{t}}z_k$$

Exercise 8. Let $f(t)$ be a T-periodic function with Fourier coefficients z_k.

- Show that $-f(t)$ has the negated Fourier coefficients $-z_k$.
- Show that $f(-t)$ has the conjugate complex Fourier coefficients $\overline{z_k}$.

- Use the above results to show that the Fourier coefficients of an even function are real and the Fourier coefficients of an odd function are imaginary.

Solution for Exercise 8. As z_k are the Fourier coefficients of $f(t)$, it holds that

$$z_k = \frac{1}{T}\int_0^T f(t)e^{-jk\omega t}dt.$$

- The Fourier coefficients of $-f(t)$ are

$$\begin{aligned}\frac{1}{T}\int_0^T -f(t)e^{-jk\omega t}dt &= -\frac{1}{T}\int_0^T f(t)e^{-jk\omega t}dt \\ &= -z_k.\end{aligned}$$

- The Fourier coefficients of $f(-t)$ are

$$\frac{1}{T}\int_0^T f(-t)e^{-jk\omega t}dt.$$

Substitution $u = -t$ gives

$$\begin{aligned}\frac{1}{T}\int_0^{-T} f(u)e^{jk\omega u}(-du) &= -\frac{1}{T}\int_0^{-T} f(u)e^{jk\omega u}du \\ &= \frac{1}{T}\int_{-T}^0 f(u)e^{jk\omega u}du.\end{aligned}$$

In order to shift the limits of integration back to $0 \dots T$ we substitute again with $v = u + T$.

$$\begin{aligned}\frac{1}{T}\int_0^T f(v-T)e^{jk\omega(v-T)}dv &= \frac{1}{T}\int_0^T f(v)e^{jk\omega v}e^{-jk\omega T)}dv \\ &= \frac{1}{T}\int_0^T f(v)e^{jk\omega v}dv \\ &= \frac{1}{T}\int_0^T f(v)\overline{e^{-jk\omega v}}dv \\ &= \overline{\frac{1}{T}\int_0^T f(v)e^{-jk\omega v}dv} \\ &= \overline{z_k}.\end{aligned}$$

- For an even function $f(t)$ it holds that

$$f(t) = f(-t).$$

Its Fourier coefficients have therefore the property

$$z_k = \overline{z_k}.$$

It follows that the imaginary part of z_k is zereo, i.e. z_k is real.

For an odd function $f(t)$ it holds that

$$f(t) \quad = \quad -f(-t).$$

Its Fourier coefficients have therefore the property

$$z_k \quad = \quad -\overline{z_k}.$$

Taking real part on both sides we obtain

$$\mathrm{re}(z_k) \quad = \quad -\mathrm{re}(z_k).$$

It follows that $\mathrm{re}(z_k) = 0$, i.e. z_k ist imaginary.

Exercise 9. Compute the complex Fourier coefficients z_k of the $T = 2$-periodic function

$$f(t) = e^{|t|} \text{ for } -1 < t \leq 1$$

and $f(t+2) = f(t)$ for all t. Simplify the term for z_k as much as possible.

Solution for Exercise 9. From $T = 2$ it follows that $\omega = \pi$. Therefore it holds that

$$\begin{aligned}
z_k &= \frac{1}{2}\int_{-1}^{1} e^{|t|}e^{-jk\omega t}dt \\
&= \frac{1}{2}\int_{-1}^{0} e^{-t}e^{-jk\omega t}dt + \frac{1}{2}\int_{0}^{1} e^{t}e^{-jk\omega t}dt \\
&= \frac{1}{2}\int_{-1}^{0} e^{(-1-jk\omega)t}dt + \frac{1}{2}\int_{0}^{1} e^{(1-jk\omega)t}dt \\
&= -\frac{1}{2+2jk\omega}[e^{(-1-jk\omega)t}]_{-1}^{0} + \frac{1}{2-2jk\omega}[e^{(1-jk\omega)t}]_{0}^{1} \\
&= -\frac{1}{2+2jk\omega}\left(1 - e^{1+jk\omega}\right) + \frac{1}{2-2jk\omega}\left(e^{1-jk\omega} - 1\right) \\
&= \frac{1}{2+2jk\pi}\left(ee^{jk\pi} - 1\right) + \frac{1}{2-2jk\pi}\left(ee^{-jk\pi} - 1\right)
\end{aligned}$$

As k is an integer it holds that

$$e^{jk\pi} = e^{-jk\pi}$$

and therefore

$$\begin{aligned}
z_k &= \frac{1}{2}\left(\frac{1}{1+jk\pi} + \frac{1}{1-jk\pi}\right)\left(ee^{jk\pi} - 1\right) \\
&= \frac{1}{1+k^2\pi^2}\left(ee^{jk\pi} - 1\right).
\end{aligned}$$

As

$$e^{jk\pi}\begin{cases} 1 & \text{if } k \text{ even} \\ -1 & \text{if } k \text{ odd} \end{cases}$$

it holds for even k

$$z_k = \frac{e-1}{1+k^2\pi^2}$$

and for odd k

$$z_k = \frac{-e-1}{1+k^2\pi^2}.$$

This can be expressed without case distinction as

$$z_k = \frac{(-1)^k e-1}{1+k^2\pi^2}.$$

Exercise 10. Compute the Fourier coefficients z_k of the T-periodic function

$$f(t) = 3+\cos(\omega t) - 4\cos(3\omega t+2) + \sin(\omega t) + 2\sin(4\omega t)$$

with $\omega = 2\pi/T$. Hint: Try to solve this exercise without integration.

Solution for Exercise 10. First we rewrite f as a sum

$$f(t) = A_0 + \sum_k A_k \cos(k\omega t + \varphi_k)$$

with $A_k \geq 0$. Comparing the summands with equal frequency we obtain

$$\begin{aligned} A_0 &= 3 \\ A_1\cos(\omega t+\varphi_1) &= \cos(\omega t)+\sin(\omega t) \\ A_3\cos(3\omega t+\varphi_3) &= -4\cos(3\omega t+2) \\ A_4\cos(4\omega t+\varphi_4) &= 2\sin(4\omega t). \end{aligned}$$

In order to determine A_k and φ_k the right hand sides are rewritten.

- $k=1$.

$$\begin{aligned} \cos(\omega t)+\sin(\omega t) &= \frac{1}{2}\left(e^{j\omega t}+e^{-j\omega t}\right) + \frac{1}{2j}\left(e^{j\omega t}-e^{-j\omega t}\right) \\ &= \frac{1}{2}\left(1+\frac{1}{j}\right)e^{j\omega t} + \frac{1}{2}\left(1-\frac{1}{j}\right)e^{-j\omega t} \\ &= \mathrm{re}\left((1-j)e^{j\omega t}\right) \\ &= \mathrm{re}\left(\sqrt{2}e^{-j\pi/4}e^{j\omega t}\right) \\ &= \sqrt{2}\mathrm{re}\left(e^{j(\omega t-\pi/4)}\right) \\ &= \sqrt{2}\cos(\omega t-\pi/4). \end{aligned}$$

  Hence

$$\begin{aligned} A_1\cos(\omega t+\varphi_1) &= \sqrt{2}\cos(\omega t-\pi/4) \\ A_1 &= \sqrt{2} \\ \varphi_1 &= -\pi/4. \end{aligned}$$

- $k = 3$. From $\cos(x) = -\cos(x - \pi)$ it follows that

$$-4\cos(3\omega t + 2) = 4\cos(3\omega t + 2 - \pi).$$

Hence

$$\begin{aligned} A_3 \cos(3\omega t + \varphi_3) &= 4\cos(3\omega t + 2 - \pi) \\ A_3 &= 4 \\ \varphi_3 &= 2 - \pi. \end{aligned}$$

- $k = 4$. From $\sin(x) = \cos(x - \pi/2)$ it follows that

$$2\sin(4\omega t) = 2\cos(4\omega t - \pi/2).$$

Hence

$$\begin{aligned} A_4 \cos(4\omega t + \varphi_4) &= 2\cos(4\omega t - \pi/2) \\ A_4 &= 2 \\ \varphi_4 &= -\pi/2. \end{aligned}$$

As

$$\begin{aligned} z_0 &= A_0 \\ z_k &= \frac{1}{2} A_k e^{j\varphi_k} \text{ f"ur } k > 0 \\ z_{-k} &= \overline{z_k} \end{aligned}$$

it holds that

$$\begin{aligned} z_0 &= 3 \\ z_1 &= \frac{1}{\sqrt{2}} e^{-j\pi/4} \\ z_{-1} &= \frac{1}{\sqrt{2}} e^{j\pi/4} \\ z_3 &= 2e^{j(2-\pi)} \\ &= -2e^{2j} \\ z_{-3} &= -2e^{-2j} \\ z_4 &= e^{-j\pi/2} \\ &= -j \\ z_{-4} &= j. \end{aligned}$$

All other Fourier coefficients are zero.

Exercise 11. Compute the complex Fourier coefficients z_k of the T-periodic sawtooth function f which is defined by

$$\begin{aligned} f(t) &= t \quad \text{for } 0 \le t < T \\ f(t + T) &= f(t) \quad \text{else} \end{aligned}$$

Solve the integrals without computer using partial integration.

Solution for Exercise 11. Let

$$\omega = \frac{2\pi}{T}.$$

Then

$$\begin{aligned} z_k &= \frac{1}{T}\int_0^T f(t)e^{-jk\omega t}dt \\ &= \frac{1}{T}\int_0^T te^{-jk\omega t}dt \end{aligned}$$

Partial integration with

$$\begin{aligned} g(t) &= t \\ h'(t) &= e^{-jk\omega t} \\ g'(t) &= 1 \\ h(t) &= -\frac{e^{-jk\omega t}}{jk\omega} \end{aligned}$$

gives

$$\begin{aligned} \frac{1}{T}\int_0^T te^{-jk\omega t}dt &= \frac{1}{T}\left(\left[-t\frac{e^{-jk\omega t}}{jk\omega}\right]_0^T + \int_0^T \frac{e^{-jk\omega t}}{jk\omega}dt\right) \\ &= \frac{1}{Tjk\omega}\left(\left[-te^{-jk\omega t}\right]_0^T + \int_0^T e^{-jk\omega t}dt\right) \\ &= \frac{1}{2\pi jk}\left(-T\underbrace{e^{-2\pi jk}}_{=1} - \left[\frac{e^{-jk\omega t}}{jk\omega}\right]_0^T\right) \\ &= \frac{1}{2\pi k^2\omega}\left(Tjk\omega + \left[e^{-jk\omega t}\right]_0^T\right) \\ &= \frac{T}{4\pi^2k^2}\left(2\pi jk + \underbrace{e^{-2\pi jk}}_{=1} - 1\right) \\ &= \frac{jT}{2\pi k} \end{aligned}$$

Due to the division by k this holds only for $k \neq 0$. For the special case $k = 0$ we obtain

$$\begin{aligned} z_0 &= \frac{1}{T}\int_0^T tdt \\ &= \frac{1}{2T}\left[t^2\right]_0^T \\ &= \frac{1}{2T}T^2 \\ &= T/2 \end{aligned}$$

Summarizing we have

$$z_k = \begin{cases} T/2 & \text{if } k = 0 \\ jT/(2\pi k) & \text{else.} \end{cases}$$

Exercise 12. Compute the complex Fourier coefficients z_k of the $T = 2$-periodic function

$$f(t) = \begin{cases} t+1 & \text{if } -1 < t < 0 \\ 1 & \text{if } 0 \leq t \leq 1 \end{cases}$$

and $f(t+2) = f(t)$ for alle t. Solve the integrals without computer with partial integration.

Solution for Exercise 12. With $T = 2$ and

$$\omega = \frac{2\pi}{T} = \pi$$

it holds that

$$\begin{aligned} z_k &= \frac{1}{T}\int_0^T f(t)e^{-jk\omega t}dt \\ &= \frac{1}{2}\int_{-1}^1 f(t)e^{-jk\pi t}dt \\ &= \frac{1}{2}\int_{-1}^0 (t+1)e^{-jk\pi t}dt + \frac{1}{2}\int_0^1 e^{-jk\pi t}dt. \end{aligned}$$

In both cases the integrals are obtained as follows:

$$\begin{aligned} \int_{-1}^0 (t+1)e^{-jk\pi t}dt &= \frac{1}{-jk\pi}\left[(t+1)e^{-jk\pi t}\right]_{-1}^0 - \frac{1}{-jk\pi}\int_{-1}^0 e^{-jk\pi t}dt \\ &= \frac{1}{-jk\pi} - \frac{1}{(-jk\pi)^2}\left[e^{-jk\pi t}\right]_{-1}^0 \\ &= \frac{j}{k\pi} + \frac{1}{k^2\pi^2}\left(1 - e^{jk\pi}\right) \\ \int_0^1 e^{-jk\pi t}dt &= \frac{1}{-jk\pi}\left[e^{-jk\pi t}\right]_0^1 \\ &= \frac{je^{-jk\pi} - j}{k\pi}. \end{aligned}$$

It follows that

$$\begin{aligned} z_k &= \frac{1}{2}\left(\frac{j}{k\pi} + \frac{1 - e^{jk\pi}}{k^2\pi^2} + \frac{je^{-jk\pi} - j}{k\pi}\right) \\ &= \frac{1}{2}\left(\frac{je^{-jk\pi}}{k\pi} + \frac{1 - e^{jk\pi}}{k^2\pi^2}\right) \\ &= \frac{je^{-jk\pi}}{2k\pi} + \frac{1 - e^{jk\pi}}{2k^2\pi^2} \\ &= \begin{cases} \frac{j}{2k\pi} & \text{if } k \text{ even} \\ \frac{-j}{2k\pi} + \frac{1}{k^2\pi^2} & \text{if } k \text{ odd} \end{cases} \end{aligned}$$

For $k = 0$ an exception is necessary because of the division by k:

$$
\begin{aligned}
z_0 &= \frac{1}{2}\int_{-1}^{1} f(t)e^{-jk\pi t}dt \\
&= \frac{1}{2}\int_{-1}^{1} f(t)dt \\
&= \frac{1}{2}\left(\int_{-1}^{0}(t+1)dt + \int_{0}^{1} 1dt\right) \\
&= \frac{1}{2}\left(\frac{1}{2}+1\right) \\
&= \frac{3}{4}.
\end{aligned}
$$

You can plot the result with matlab with the following code:

```
syms k zk f t;
zk = (j*exp(-j*k*pi)) / (2*k*pi) + (1-exp(j*k*pi)) / (2*k*k*pi*pi);
f = 2*real(symsum(zk*exp(j*k*pi*t),k,1,30)) + 3/4;
ezplot(f,[-2,2]);
```

Exercise 13. Compute the complex Fourier coefficients z_k of the π-periodic function

$$f(t) = \sin(t)^2.$$

Hint: Rewrite $f(t)$ as a sum of complex exponentials.

Solution for Exercise 13. From the period duration $T = \pi$ it follows that $\omega = 2$.

$$
\begin{aligned}
f(t) &= \left(\frac{1}{2j}(e^{jt} - e^{-jt})\right)^2 \\
&= -\frac{1}{4}\left(e^{j2t} - 2 + e^{-j2t}\right)
\end{aligned}
$$

- Fourier coefficients for e^{j2t}. For $k \neq 1$ it holds that

$$
\begin{aligned}
z_k &= \frac{1}{\pi}\int_{0}^{\pi} e^{j2t}e^{-jk2t}dt \\
&= \frac{1}{\pi}\int_{0}^{\pi} e^{j2(1-k)t} \\
&= \frac{1}{2\pi j(1-k)}\left[e^{j2(1-k)t}\right]_0^{\pi} \\
&= \frac{1}{2\pi j(1-k)}\left(e^{j2(1-k)\pi} - 1\right) \\
&= 0
\end{aligned}
$$

For $k = 1$ we obtain

$$
\begin{aligned}
z_1 &= \frac{1}{\pi}\int_0^{\pi} e^{j2t}e^{-j2t}dt \\
&= \frac{1}{\pi}\int_0^{\pi} 1dt \\
&= 1.
\end{aligned}
$$

- Fourier coefficients for e^{-j2t}. For $k \neq -1$ it holds that

$$
\begin{aligned}
z_k &= \frac{1}{\pi}\int_0^{\pi} e^{-j2t}e^{-jk2t}dt \\
&= \frac{1}{\pi}\int_0^{\pi} e^{-j2(1+k)t} \\
&= \frac{1}{-2\pi j(1+k)}\left[e^{-j2(1+k)t}\right]_0^{\pi} \\
&= \frac{1}{-2\pi j(1+k)}\left(e^{-j2(1+k)\pi} - 1\right) \\
&= 0
\end{aligned}
$$

For $k = -1$ we obtain

$$
\begin{aligned}
z_{-1} &= \frac{1}{\pi}\int_0^{\pi} e^{-j2t}e^{j2t}dt \\
&= \frac{1}{\pi}\int_0^{\pi} 1dt \\
&= 1.
\end{aligned}
$$

- Fourier coefficients of the constant -2 function are

$$
z_k = \begin{cases} -2 & \text{if } k = 0 \\ 0 & \text{else.} \end{cases}
$$

The Fourier coefficients of $\sin(t)^2$ are therefore

$$
z_k = \begin{cases} 1/2 & \text{if } k = 0 \\ -1/4 & \text{if } k = 1 \text{ or } k = -1 \\ 0 & \text{else.} \end{cases}
$$