Heilbronn, 30.9.2025 Prof. Dr. V. Stahl WS 25/26

Homework for Digital Signal Processing

Sheet 1

Exercise 1. The convolution $f * g$ of two functions f, g ist defined by

$$(f * g)(t) \quad = \quad \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau.$$

Let $f \in \mathbb{R} \to \mathbb{R}$ und $c \in \mathbb{R}$. The function cf ist defined by

$$cf \in \mathbb{R} \to \mathbb{R}, \quad (cf)(t) = cf(t).$$

Show that

$$(cf) * g \quad = \quad c(f * g).$$

Exercise 2. The delay of a signal f by $\hat{t}$ is indicated by an index $\hat{t}$, i.e.

$$f_{\hat{t}}(t) \quad = \quad f(t - \hat{t}).$$

Show that

$$f_{\hat{t}} * g \quad = \quad (f * g)_{\hat{t}}.$$

Exercise 3. Let g be a T-periodic function. Show that $f * g$ is also a T-periodic function for any function f.

Exercise 4. Let $a \leq b$ and

$$g(t) \quad = \quad \begin{cases} 1 & \text{if } a \leq t \leq b \\ 0 & \text{else.} \end{cases}$$

Show that for any function f it holds that

$$(f * g)(t) \quad = \quad \int_{t-b}^{t-a} f(x)dx.$$

Exercise 5. Show that convolution is commutative, i.e. for all functions f, g it holds that

$$f * g = g * f.$$

Exercise 6. For $a > 0$ let index a denote the compression of a function by factor a, i.e.

$$f_a(t) \quad = \quad f(at).$$

Show that

$$f_a * g_a \quad = \quad \frac{1}{a}(f * g)_a.$$

Exercise 7. Let $f, g, h \in \mathbb{R} \to \mathbb{R}$ be functions. Function $g + h$ ist defined by

$$g + h \in \mathbb{R} \to \mathbb{R}, \quad (g + h)(t) = g(t) + h(t).$$

Show that

$$f * (g + h) = (f * g) + (f * h).$$

Exercise 8. Show that $\sigma * f$ is an antiderivative of f, i.e.

$$(\sigma * f)' \quad = \quad f.$$

This means that convolution with σ causes integration.

Exercise 9. Show that

$$(f * g)' \quad = \quad f * g'.$$

The proof is straight forward with Fourier Transform but it is a good exercise to do it in time domain as well.

Exercise 10. The Heaviside step function $\sigma(t)$ ist defined by

$$\sigma(t) \quad = \quad \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{else.} \end{cases}$$

- Let

  $$\begin{aligned} f(t) &= \sigma(t)e^{at} \\ g(t) &= \sigma(t)e^{bt}. \end{aligned}$$

  Compute

  $$(f * g)(t)$$

  and simplify the result as much as possible. Consider also the special case $a = b$.

- Use your result to compute $f * g$ for

  $$\begin{aligned} f(t) &= \sigma(t)\sin(t) \text{ and} \\ g(t) &= \sigma(t)\cos(t). \end{aligned}$$

  Hint:

  $$\sin(t) \quad = \quad \frac{1}{2j}\left(e^{jt} - e^{-jt}\right).$$