Homework for Digital Signal Processing

Sheet 2

Exercise 1. Implement the following functions (matlab is easiest):

- A function which reads a sound file (WAV format) and writes the samples (one channel in case of stereo) to a text file.
- A function which reads samples from a text file and stores the result in a sound file (WAV format, mono, 8kHz sampling rate).
- A function which plots the samples of a sound file.

If you decide to do your project in C, you can leave the sound file I/O to matlab and read/write samples from/to simple text files. It is also not difficult to implement sound file I/O directly in C if you make yourself familiar with the WAV header and ask Chat GPT for an example.

Finally write a program in the language for your project which generates samples for a sine wave with 200Hz and 8kHz sampling rate and duration one second. Store the samples in a sound file in WAV format. Play the sound file and verify that it sounds correct. Try different frequencies and see what happens if the frequency is above 4kHz.

Exercise 2. Find a complex number z such that

$$3\cos(2t+1) = \mathrm{re}(ze^{j\,2t}).$$

Exercise 3. Find $a, b \in \mathbb{R}$ such that

$$3\cos(2t+1) = a\cos(2t) + b\sin(2t).$$

Exercise 4. A function $f(t)$ is T-periodic if

$$f(t+T) = f(t) \text{ for all } t.$$

Show that

$$f(t) = e^{j\omega t} \text{ with } \omega = \tfrac{2\pi}{T}$$

is a T-periodic function.

Exercise 5. Show that the Fourier coefficients z_k of a T-periodic function $f(t)$ can also be computed with the following formula:

$$z_k = \int_0^1 f(Tt)e^{-2\pi jkt}dt.$$

Show that the function

$$g(t) = f(Tt)$$

which appears in this formula, is 1-periodic.

Exercise 6. Compute the complex Fourier coefficients z_k of the T-periodic function f, defined as

$$f(t) = \begin{cases} 0 & \text{for } -T/2 < t < 0 \\ 1 & \text{for } 0 \le t \le T/2 \end{cases}$$

and $f(t+T) = f(t)$ for all t. Solve the integrals without a computer. Hint: You have to distinguish the cases k even and k odd.

Exercise 7. Let $f(t)$ be a T-periodic function with given Fourier coefficients z_k. The function

$$g(t) = f(t - \hat{t})$$

is $f(t)$ shifted by $\hat{t}$ to the right and therefore also T-periodic. Compute the Fourier coefficients of $g(t)$ in dependence of z_k.

Exercise 8. Let $f(t)$ be a T-periodic function with Fourier coefficients z_k.

- Show that $-f(t)$ has the negated Fourier coefficients $-z_k$.
- Show that $f(-t)$ has the conjugate complex Fourier coefficients $\overline{z_k}$.
- Use the above results to show that the Fourier coefficients of an even function are real and the Fourier coefficients of an odd function are imaginary.

Exercise 9. Compute the complex Fourier coefficients z_k of the $T = 2$-periodic function

$$f(t) = e^{|t|} \text{ for } -1 < t \le 1$$

and $f(t+2) = f(t)$ for all t. Simplify the term for z_k as much as possible.

Exercise 10. Compute the Fourier coefficients z_k of the T-periodic function

$$f(t) = 3 + \cos(\omega t) - 4\cos(3\omega t + 2) + \sin(\omega t) + 2\sin(4\omega t)$$

with $\omega = 2\pi/T$. Hint: Try to solve this exercise without integration.

Exercise 11. Compute the complex Fourier coefficients z_k of the T-periodic sawtooth function f which is defined by

$$\begin{aligned} f(t) &= t && \text{for } 0 \le t < T \\ f(t+T) &= f(t) && \text{else} \end{aligned}$$

Solve the integrals without computer using partial integration.

Exercise 12. Compute the complex Fourier coefficients z_k of the $T = 2$-periodic function

$$f(t) = \begin{cases} t+1 & \text{if } -1 < t < 0 \\ 1 & \text{if } 0 \le t \le 1 \end{cases}$$

and $f(t+2) = f(t)$ for alle t. Solve the integrals without computer with partial integration.

Exercise 13. Compute the complex Fourier coefficients z_k of the π-periodic function

$$f(t) = \sin(t)^2.$$

Hint: Rewrite $f(t)$ as a sum of complex exponentials.