Homework for Digital Signal Processing

Sheet 8

Exercise 1. Implement a function for

- non-cyclic convolution
- cyclic convolution in time domain and
- cyclic convolution via FFT.

Compare the computing time and the results of cyclic convolution in time- and frequency domain. Use vectors with *integer* components for your tests. The result of the convolutions have to be integers again. Due to rounding errors the results may deviate slightly. However, if you round them to integers, the results have to be identical.

Exercise 2. Let $\vec{g}, \vec{f}, \vec{F} \in \mathbb{C}^n$ with

$$f_\ell \quad \circ\!\!-\!\!\bullet \quad F_k$$

and

$$g_\ell \quad = \quad f_{(\ell+1) \bmod n}, \qquad \ell = 0, \ldots, n-1.$$

Show that

$$g_\ell \quad \circ\!\!-\!\!\bullet \quad e^{2\pi jk/n} F_k.$$

Exercise 3. Let f, g be two signals of length F and G, i.e.

$$\begin{aligned} f &= \langle f_0, f_1, \ldots, f_{F-1} \rangle \\ g &= \langle g_0, g_1, \ldots, g_{G-1} \rangle \end{aligned}$$

and $F \geq G$.

The signals are packed into vectors with F components each, where g is zero padded with $F - G$ zeros:

$$\vec{f} = \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_{F-1} \end{pmatrix}, \qquad \vec{g} = \begin{pmatrix} g_0 \\ g_1 \\ \vdots \\ g_{G-1} \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Show that for all ℓ with $G - 1 \leq \ell \leq F - 1$ it holds that

$$(f * g)_\ell \quad = \quad (\vec{f} \otimes \vec{g})_\ell.$$